[NEW VOICES]

WHAT DO THE SIGNS SAY?

Learning to teach mathematics notation

BRYONY HEATH discusses the importance of establishing meaning for mathematical notation rather than simply focusing on naming symbols.

As students progress through primary school, they are introduced to a growing collection of notation and symbols. If students cannot see a meaningful relationship between these signs and symbols and mathematical concepts, coming to grips with notation can be difficult. This matter was highlighted for me during a recent pre-service professional experience with a Year 3 class, when I planned and taught an introductory lesson on the symbols “is less than” (<) and, “is greater than” (>). In this article I want to explore some teaching and learning concerns around introducing formal mathematical notation. I hope that sharing and reflecting upon my own experience may also provide some insights for fellow teachers trying to make notation meaningful in their primary mathematics classrooms.

What is the big idea?

From a teaching perspective, before introducing new mathematical symbols it is important to consider the role they play. Although it may not always seem that way, their purpose is to make complicated mathematical concepts more tangible and accessible. Formal notation allows mathematical thinking to be read, written and discussed with greater ease. Briefly put, it constitutes a technical shorthand

for representing the ideas and procedures of mathematics. However, not only does the use of formal notation allow students efficiently to record and share their mathematical thinking, it also provides them with the tools for moving beyond finite absolutes and developing a richer algebraic sense of number. Mathematics notation is not merely a system for reading and writing, it is a way of thinking.

Research, however, has shown that the manner in which signs and symbols are presented can lead to the frequent perception among children that they are instructions or demands (Behr, Erlwanger & Nichols, 1980); an interpretation encouraged by too great an emphasis on simple and traditional number sentences. For example, the empty box for the answer on the right of the equal sign in 2 + 3 = □ suggests to children that "=" means "Write the answer in this box." The understanding that "=" represents equivalence can be developed if the format of number work is varied, as in: □ = 2 + 3 or □ + 3 = 5. The equal sign provides just one instance of the need for number work to be presented in ways that support students' development of a flexible and resilient number sense (Bobis, Mulligan & Lowrie, 2009, p.183). This is very important in early mathematics because without number sense children become reliant on memorisation and formulae, and struggle with unfamiliar problems and the progression to more abstract mathematics such as algebra.

Helping to establish notation as a mathematical tool, rather than a set of instructions, also scaffolds students' abilities to choose and use appropriate notation confidently. This is important when they are faced with number work such as identifying patterns and problem solving, where students must decide upon which mathematical process to employ.

From this examination of the role of notation in mathematics, it can be seen how important it is to equip students with understanding of the mathematical symbols that they read and write. As in the example of the equal sign, teaching children how to use symbols, rather than emphasising what they mean, can prove counter-productive in the long run. By helping students see generalisable relationships in their understanding of number and notation, they are able to move towards abstract and algebraic thinking (Radford, 2000).

> OR <

Greater or lesser?

When I began planning my lesson on these symbols during my professional experience, I was warned by my co-operating teacher that this pair could be particularly troublesome. It is easy to see how this could be so: although they represent opposing statements they look very similar, unlike other pairs of symbols such as + and – or × and ÷. So, if children cannot differentiate the two, they are left wondering which way "the arrow" ought to point and may take a fifty-fifty guess. I found that most of the exercises in the class textbook took the form of "a □ b," with the correct sign either ">" or "<" to go in the empty square. Like many textbook questions, the focus was on simple and repetitive procedures (Vincent & Stacey, 2008). The format is designed for children to familiarise themselves with using the symbols. Of course the effectiveness of textbook work always relies very much on the teacher's approach. I felt that, without a thoughtful preamble, these exercises would prove boring for those who understood the symbols and unhelpful for those who did not.

Moreover the textbook's explanation of these symbols, like the formal name for the notation "is greater than" (>) and "is less than" (<), relied on a left-to-right reading of the number sentence. For instance, "10 > 3" represents "10 is greater than 3," however, based on the statement "3 < 10," it is still true to observe that "10 is greater than 3"—as someone who reads right-to-left might.

However, if you teach this person that, no matter what, “>” means “is greater than” and they continue to read the numbers right-to-left, they might produce this statement: “3 > 10.” When the meaning of the symbol is lost for the sake of naming the symbol, as in the last statement, the symbol becomes useless.

Meeting the diverse language needs of students is a prominent concern for primary teachers, particularly given the increasing proportion of students from non-English speaking backgrounds (NESB). The high number of NESB students in my class, 21 out of 31, was a crucial factor that led me to consider carefully how I would approach my lesson on the “>” and “<” symbols.

Practicing in symbolism

I planned and taught my lesson on the symbols “>” and “<” based on the understanding that children need to visualise and represent numerical concepts in ways that are meaningful to them. In order to assimilate new information learners need to recognise the all important connections between external representations and their own ideas (Bobis, Mulligan & Lowrie, 2009, p. 182). With that in mind, I began by drawing the pictorial representation on the board shown in Figure 1.

Figure 1. Pictorial representation.

I asked the class what was happening in the picture and received these responses:

“There are trees.”

“Different sized trees.”

I probed for a bit more description and students offered:

“The trees are getting smaller.”

“The trees are getting bigger.”

Depending which way you “read” the trees, both statements are correct and the crucial point was that the students had quickly decoded my visual representation of a sequential change in size.

I then added an outline to the diagram as shown in Figure 2.

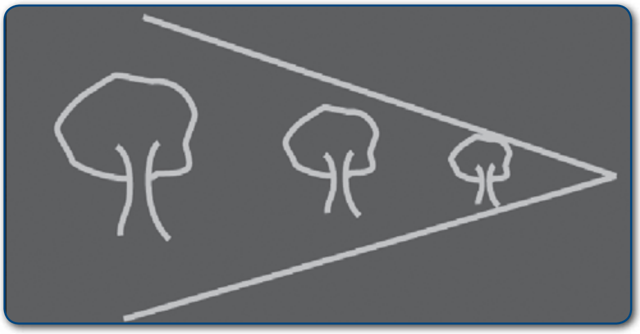

Figure 2. Pictorial representation with outline.

As a class, we discussed the relationship between the shape of the outline (open end tapering to a closed point) and the increasing/decreasing tree size.

Next, I rubbed off the middle tree and asked the class to think about what symbol we could use to describe the difference between the small tree and the large tree. I prompted students to think about the outline and soon had suggestions for using a small version as our symbol (see Figure 3).

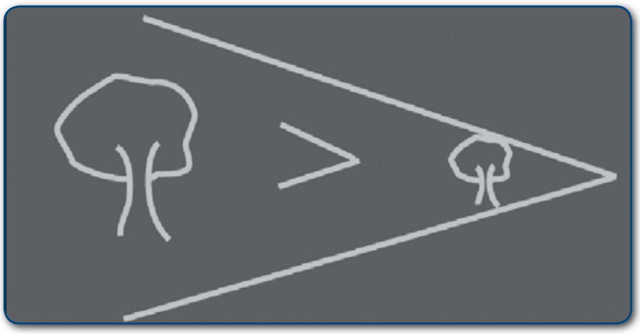

Figure 3. Incorporation of “>” sign.

I drew another set of pictures that increased in the other direction, rubbed out the middle one again and asked if we could put the same symbol in. I was very promptly told that we could but it needed to face the other way.

So now we had two pictorial diagrams

using “greater than” and “less than” symbols. Given the high language needs in the class I had decided it would be counter-productive to introduce the vocabulary of “greater than” and “less than” before clarifying the number concepts.

I rubbed the trees off the board and put in a number where the small tree had been; e.g:

$\boxed{\_ > 37}$

At first students suggested only 38 (the next number in the sequence of counting numbers) or 39 (interpreting the “>” as a missing number). Gradually though, by emphasising that there were many correct answers, I was able to elicit lots of different responses about what number could go where the large tree had been. For instance, it could be 38, just one greater than 37, or it could be 71 greater, or 100 greater. The pictorial representation had built a clear understanding of the symbols as descriptors of a size or value relationship between two numbers. The intent of the discussion and numeric examples was to establish that the value relationship the symbols described was non-exact. In addition, understanding the representation I had provided did not rely on reading left to right.

By the end of the lesson I doubt whether my students could have named “<” as “is less than” and “>” as “is greater than,” but they understood the relationship that these symbols denote between numbers.

Learning to teach mathematics

Since teaching that lesson I have finished my Master of Teaching (Primary) degree. I have come away from my studies and professional experience with a lot of ideas and twice as many questions about teaching mathematics.

In reflecting upon this lesson, I could not help but ponder what I would do differently next time: I would use more than three pictures in my initial diagram to avoid a skip count interpretation of the greater than and less than symbols; I would allow students time to explore and discuss their own ideas for greater than and less than symbols before providing the formal signs; and so on.

However, those are just details, because what that teaching experience has underscored for me is the crucial intersection of mathematical notation and number sense. Now that I understand the importance of scaffolding students’ recognition of this relationship, I feel I have the right perspective to learn to teach notation effectively.

Speaking more generally, reflecting on this lesson has consolidated a guiding principle for my mathematics teaching practice going forward. If it is more important that students understand that “=” means “the same as” than to be able to call it an equal sign, then it is more meaningful for a student to recognise that the value at the closed end of “<” will be less than the value at the open end than to name the symbol. In prioritising students’ mathematical understanding over mathematical knowledge, my aim is to not be a mathematics teacher but a teacher of mathematicians.

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